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COMPUTING VERTICAL PROFILE OF TEMPERATURE IN THE SOUTH-CHINA SEA USING CUBIC SPLINE FUNCTIONS

Pham Hoang Lam, Ha Thanh Huong, Pham Van Huan

University of natural sciences, VNU

Abstract: In this text the spline approximation was applied to the empirical vertical profiles of oceanographic parameters such as temperature, salinity or density to obtain a more precious and reliable result of interpolation. Our experiments with the case of observed temperature profiles in the East sea show that the cubic polynomial spline method has a higher reliability and precision in comparison with the linear interpolation and other traditional methods. The method was realized into a subroutine in our programs of management and manipulation of oceanographic data.

As an application, the observed temperature field from World Ocean Data Base 2001 consisting of about 137000 vertical profiles have been analyzed to examine the features of the vertical distribution of temperature in the East sea.

It is found that the upper homogeneous layer in the summer months is only a thin one with the thickness of about 10 m, but in the winter months this layer expands to the depth of about 50-60 m and even more. And the thickness of upper mixing layer changes largely from year to year as well with a range from about 20 m to about 70 m.

Temperature is always an important factor in the research of physics in general and particular in oceanography. With the rapid development of the information technology, the computation and prediction of the oceanographic parameters are of special interest. Sea water temperature is an important part of the input of the modern thermo-dynamical model. In many application, the water temperature and other oceanographic parameters at different horizons are required to be calculated from their observed profiles by the interpolation procedures. The spline method of approximation appears to be a reliable and precious one for these purposes (Belkin I. M. et all, 1982; Belkin I. M., 1986a, 1986b; Belkin I. M., 2001).

The purpose of the cubic spline function method is to find a cubic polynomial on each interval on a given coordinate line, in our case, is the z-coordinate of depth. Suppose that on the interval [a, b] of the z-coordinate we have a computation grid $a = z_0 < z_1 < ... < z_n = b$. At each knot, the values of the temperature function $T(z)$ at the layer which have been measured [2-5] are given by $\{T^k\}_{k=0}^n$. The interpolation and extrapolation problem using piece-wise cubic functions is to find a function $f(z)$ which satisfy the following conditions (Schoenberg I. J., 1964):

- $f(z)$ belongs to $C^2(a,b)$, that is continuous together with its first and second derivatives.

- On each interval $[z_{k-1}, z_k]$, the function $f(z)$ is a cubic polynomial of the form:

$$f(z) = f_k(z) = \sum_{l=0}^{3} a_l^{(k)} (z - z_k)^l,$$
$$k = 1, 2, ..., n. \qquad (1)$$

- Conditions at the knot of the grid:

$$f(z_k) = T_k,\ k = 0, 1, ..., n \qquad (2)$$

- The second derivative $f''(z)$ satisfies the conditions:

$$f''(a) = f''(b) \qquad (3)$$

This problem leads to a problem of solving a system of linear equations of the coefficients $a_2^{(k)}$, $(k = 0, 1, ..., n)$:

$$h_k a_2^{(k-1)} + 2(h_k + h_{k+1}) a_2^{(k)} + h_{k+1} a_2^{(k+1)} = f(k),$$
$$k = 1, 2, ..., n-1, \qquad (4)$$

where

$$a_2^{(0)} = 0, \qquad a_2^{(n)} = 0, \qquad (5)$$

$$F_k = 3\left[\frac{T_{k-1} - T_k}{h_k} - \frac{T_k - T_{k+1}}{h_{k+1}}\right],$$
$$k = 1, 2, ..., n \qquad (6)$$

and

$$h_k = x_k - x_{k-1}. \qquad (7)$$

The remaining coefficients of the system (1) are determined from the following:

$$a_0^{(k)} = T_k \qquad (8)$$

$$a_1^{(k)} = -\frac{h_k}{3}\left(a_2^{(k-1)} + 2a_2^{(k)}\right) + \frac{T_{k-1} - T_k}{h_k} \qquad (9)$$

$$a_3^{(k)} = \frac{a_3^{(k-1)} - a_2^{(k)}}{3h_k} \qquad (10)$$

The solution of the problem is assumed to be exist and unique. The main difficulty in the setting up of the interpolation problem using spline function is to find the right boundary conditions. In the interpolation problem using data from the hydrological stations, the boundary condition (3) is quite suitable with the physical environment.

To fulfill the experiments with the spline method we use the observed profiles of water temperature in the South-china sea in the database World Ocean Atlas 2001.

The temperature field is given for the horizons 0, 10, 20, 30, 50, 75, 100, 125, 150, 200, 250, 300, 400, 500, 600, 800 and 1000 m.

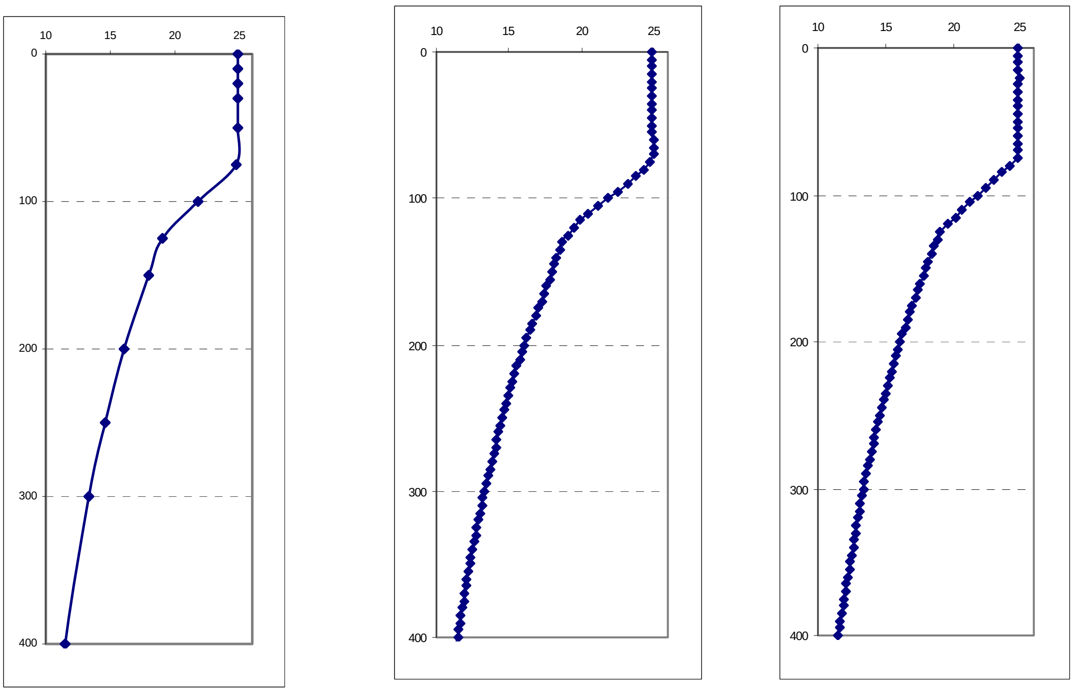
Using the cubic spline functions we have computed the temperature values from the surface layer to the 1000 m layer at different layer of distance 5 m will gives us the cubic polynomials at the intervals $[z_0, z_1]$, $[z_1, z_2]$, ..., $[z_{n-1}, z_n]$. For the vertical profile of temperature at the point of latitude 13° N and longitude 110° E, the computed coefficients of the polynomial for each of 16 depth intervals are listed in the table 1.

From these polynomials one can compute the values of the temperature at any layer through the system of coefficients a_0, a_1, a_3.

From the comparing two methods, the traditional linear interpolation and the interpolation using cubic spline functions, we can see the advantage of the later one. The cubic spline functions give smoother curve of profiles and the profiles reflect better the variation characteristics of temperature at different depth (fig. 1).

Table 1: Values of the coefficients of the cubic spline function at the dividing point at different depths

a_0	a_1	a_2	a_3
24.88	-0.000853	0.000128	-0.000004
24.89	-0.000014	-0.000212	0.000011
24.87	0.003910	-0.000181	-0.000001
24.87	-0.011432	0.000948	-0.000019
24.77	0.059762	-0.003820	0.000064
21.80	0.138229	0.000744	-0.000061
19.05	0.072143	0.001899	-0.000015
17.98	0.031601	-0.000278	0.000029
16.07	0.037510	0.000160	-0.000003
14.59	0.026389	0.000017	0.000001
13.34	0.023050	0.000050	0.000000
11.50	0.014124	0.000039	0.000000
10.24	0.011778	-0.000007	0.000000
9.05	0.011425	0.000011	0.000000
7.37	0.004491	0.000024	0.000000
6.72	0.001652	0.000000	0.000000



a) b) c)

Fig. 1. Vertical distribution of temperature at point 13° N-110° E
a) measured, b) cubic spline method, c) linear interpolation

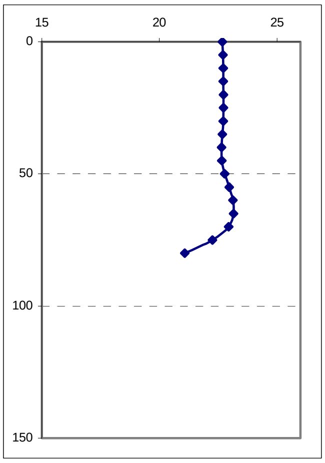


Fig. 2. Vertical distribution of temperature (22° N-116° E)

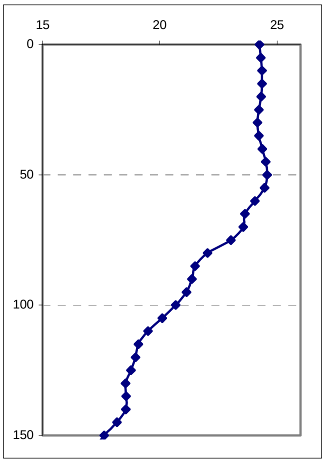


Fig. 3. Vertical distribution of temperature (19° N-112° E)

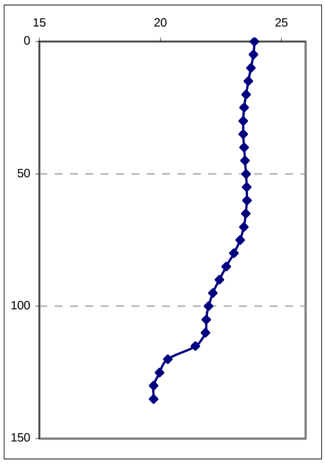


Fig. 4. Vertical distribution of temperature (16° N-109.5° E)

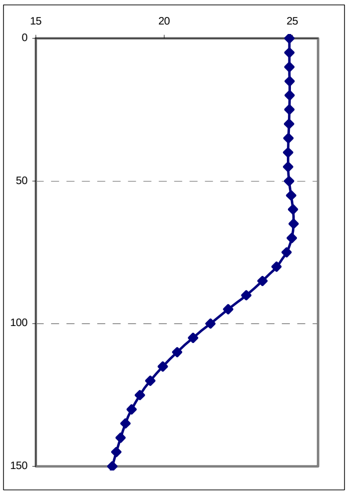


Fig. 5. Vertical distribution of temperature (13° N - 110° E)

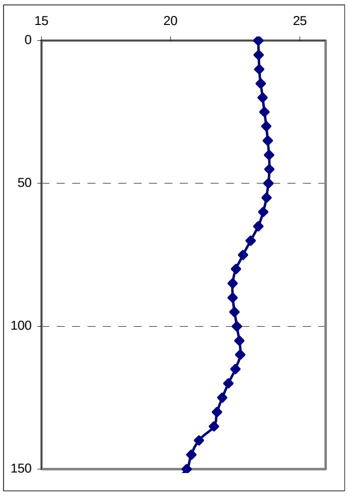


Fig. 6. Vertical distribution of temperature (10° N - 109.5° E)

Table 2. The seasonal changes of the homogeneous layer in 1966

at point 109° E - 17° N												
Month	1	2	3	4	5	6	7	8	9	10	11	12
Thickness (m)	62	60	40	10	10	15	15	–	22	50	60	60
at point 114° E - 13° N												
Month	1	2	3	4	5	6	7	8	9	10	11	12
Thickness (m)	60	65	66	45	20	–	30	30	50	40	–	–
at point 109° E - 11° N												
Month	1	2	3	4	5	6	7	8	9	10	11	12
Thickness (m)	25	–	–	–	10	8	5	–	15	30	50	–

Figures 2 to 6 show the computed profiles of some other points in the East sea as the examples.

In general, temperature tends to decrease as the depth increases. However the analysis of the vertical profile of temperature at these points shows the existence of the strongly mixed layers. At these points, the temperature is quite homogeneous, the strong mixing even makes the temperature at some layers higher than the surface temperature. These points belong to the mainly stream area, the current speed can be as high as 1m/s at surface, so the sea water will be mixed up strongly. The thickness of this mixing layer is often about 50-70 m. Under this mixing layer is the layer with the strong variation in temperature. The temperature begins to decrease fast until 150-200 m and after that it decreases gradually to the bottom. This is also the common law of changing of temperature of sea water with depth.

Base on the analyzed vertical profiles of temperature we can evaluate the variability of the upper homogeneous layer (table 2). It is clear that in the summer months the upper homogeneous layer is only a thin one with the thickness of about 10 m, in the winter months - this layer stretches to the depth of about 50-60 m and even more.

The changes of the thickness of the homogeneous layer between the years can be seen by comparison the analyzed vertical profiles at a point in winter in some years (table 3).

Table 3. The changes of the winter homogeneous layer thickness between years at point 112° E - 12° N

Year	1966	1969	1972	1980	1982	1989
Thickness (m)	66	38	40	50	22	65

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SỬ DỤNG HÀM SPLINE BẬC BA ĐỂ TÍNH TRẮC DIỆN THẲNG ĐỨNG CỦA NHIỆT ĐỘ NƯỚC BIỂN ĐÔNG

Phạm Hoàng Lâm, Hà Thanh Hương, Phạm Văn Huấn
Trường Đại học Khoa học Tự nhiên, ĐHQG Hà Nội

Xấp xỉ spline bậc ba được áp dụng đối với các trắc diện thẳng đứng thực nghiệm của các tham số hải dương học để nhận được kết quả nội suy chính xác và tin cậy hơn. Thí nghiệm của chúng tôi cho thấy rằng phương pháp spline đa thức bậc ba có độ tin cậy và chính xác hơn so với phương pháp nội suy tuyến tính. Phương pháp đã được hiện thực hóa thành thủ tục trong các chương trình quản lý và thao tác dữ liệu hải dương học của chúng tôi.

Với tư cách ứng dụng phương pháp, các trắc diện nhiệt độ thẳng đứng quan trắc lấy từ cơ sở dữ liệu nhiệt độ nước biển Đông trong World Ocean Data Base 2001 gồm 137000 trắc diện thẳng đứng nhiệt độ đã được phân tích để xem xét đặc điểm phân bố nhiệt độ thẳng đứng của vùng biển biến đổi trong năm và giữa các năm.

Thấy rằng lớp đồng nhất nhiệt độ phía trên của biển trong các tháng mùa hè chỉ là một lớp mỏng dày khoảng 10 m, nhung trong các tháng mùa đông lớp này mở rộng tới độ sâu 50-60 m và thậm chí hơn. Độ dày của lớp này cũng biến đổi mạnh từ năm này tới năm khác với dải biến thiên từ 20 m tới 70 m.

Địa chỉ liên hệ: Phạm Văn Huấn
334, Nguyễn Trãi, Thanh Xuân, Hà Nội
Điện thoại: 854945, 0912 116 661